

Bellwork: Write the series in sigma notation

$$10 + 13 + 16 + \dots + 28$$

$$\sum_{k=1}^7 3k + 7$$

$$3k + 7 = 28$$

$$3k = 21$$

$$k = 7$$

10) $1 + 4 + 9 + 16 + 25 + 36 + 49$
 $1^2 \quad 2^2 \quad 3^2 \quad 4^2 \quad 5^2 \quad 6^2 \quad 7^2$

$$\sum_{k=1}^7 k^2$$

12) $\frac{3}{4} + \frac{4}{9} + \frac{5}{16} + \dots + \frac{9}{64}$
 $\begin{matrix} a_1 \\ k=1 \end{matrix}$ $\begin{matrix} +1 \\ \rightarrow \end{matrix}$ $\begin{matrix} 4 \\ 3^2 \end{matrix}$ $\begin{matrix} 9 \\ 3^2 \end{matrix}$ $\begin{matrix} 16 \\ 4^2 \end{matrix}$ $\begin{matrix} 64 \\ 8^2 \end{matrix}$

$$\sum_{k=1}^7 \frac{k+2}{(k+1)^2}$$

Homework 9.1 Solutions

Lesson 9.2 Objectives

I can find the sum of geometric and arithmetic series

A **geometric** series is the sum of the terms of a geometric sequence, $S = \underline{a} + \underline{ar} + ar^2 + ar^3 + \dots + ar^n + \dots$, where a is the first term and r is the common ratio.

$$3 + 6 + 12 + 24 + 48 + 96$$

$$a = 3$$

$$r = 2$$

$$n = 6$$

An **arithmetic** series is the sum of the terms of an arithmetic sequence, $a_n = a_1 + (n-1)d$, where a_1 is the first term and d is the common difference.

$$2 + 6 + 10 + 14 + 18$$

$$a_1 = 2$$

$$n = 5$$

$$d = 4$$

Sum of a finite geometric series:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

where a is the first term, r is the common ratio and n is the number of terms

$$S_n = \frac{a(1-r^n)}{1-r}$$

Evaluate the sum of the finite geometric series

$$1+3+9+27+81+243+729$$

$$a = 1$$

$$r = 3$$

$$n = 7$$

$$S = \frac{a(1-r^n)}{1-r}$$

$$S = \frac{1(1-3^7)}{(1-3)} = 1093$$

$$2. \sum_{k=1}^{10} 5 \left(\frac{1}{2}\right)^{k-1} = 5 + 5\left(\frac{1}{2}\right) + 5\left(\frac{1}{2}\right)^2 + \dots$$

$$r = \frac{1}{2}$$

$$a_1 = 5$$

$$n = 10$$

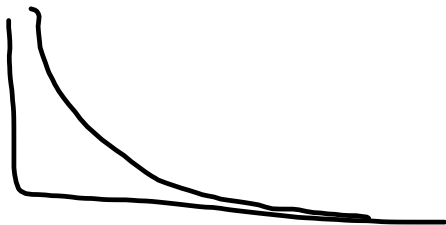
$$+ 5\left(\frac{1}{2}\right)^9$$

$$S = \frac{5(1 - (\frac{1}{2})^{10})}{(1 - \frac{1}{2})} = \boxed{9.99}$$

Sum of an infinite geometric series:

$$S = \frac{a}{1-r} \quad \text{if } |r| \geq 1, \text{ sum is } \infty$$

$$\text{if } |r| < 1, \text{ use formula}$$



$$3. \frac{5}{6} + \frac{25}{36} + \frac{125}{216} + \dots$$

$$S = \frac{a}{1-r} = \frac{5/6}{(1-5/6)}$$

$$a = \frac{5}{6}$$

$$r = \frac{5}{6} < 1$$

$$= \boxed{5}$$

$$4. \sum_{k=1}^{\infty} \left(\frac{4}{\pi}\right)^k = \frac{4}{\pi} + \left(\frac{4}{\pi}\right)^2 + \left(\frac{4}{\pi}\right)^3$$

$$a = \frac{4}{\pi}$$

$$r = \frac{4}{\pi} > 1$$

$$\boxed{\infty}$$

Sum of a finite arithmetic series: $a_1 + a_2 + a_3 + \dots$

$a_2 - a_1$
 $a_3 - a_2$

$$S_n = \frac{n}{2}(a_1 + a_n) \quad \text{or} \quad S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

\uparrow
last-term
 \uparrow
Common
difference

$$\frac{a_2}{a_1} = \frac{a_3}{a_2}$$

5. $-5 - 11 - 17 - 23 - \dots - 71$

$a_1 = -5$
 $a_n = -71$
 $n = 12$

$$S = \frac{n}{2}(a_1 + a_n) = \frac{12}{2}(-5 + -71) \quad d = -6$$

$$\sum_{k=1}^{12} -6k + 1 \quad 6(-76) = \boxed{-456}$$

$$6) \sum_{k=1}^8 4k+3$$

$$\begin{aligned} n &= 8 \\ a_1 &= 7 \\ a_n &= 35 \end{aligned}$$

$$S = \frac{n}{2}(a_1 + a_n) = \frac{8}{2}(7 + 35) = \boxed{168}$$

$$S = \frac{n}{2}(a_1 + (n-1)d) = \frac{8}{2}(2(7) + (7)4) = \boxed{168}$$

7. A professional baseball player signs a contract with a beginning salary of \$2,250,000 for the first year and an annual increase of 5% per year beginning in the second year. How much money in total will the athlete make if his contract is for 6 years? Round to the nearest dollar.

$$S = \frac{a(1-r^n)}{(1-r)}$$

$$a = \underline{2,250,000}$$

$$n = 6 \text{ yrs}$$

$$r = 1.05$$

$$\frac{2250000(1-1.05^6)}{(1-1.05)} = 15,304,304$$

