

## COMBINING \& COMPARING FUNCTIONS

## OBJECTIVE

1. SWBAT build new functions by combining functions.
2. SWBAT compare key features of functions.

## Combining Functions

Functions can be added, subtracted, multiplied or divided to form new functions. Consider the following examples for the functions $a(x)=x^{2}+2 x$ and $b(x)=7 x-5$
$a(x)+b(x)=x^{2}+2 x+7 x-5=x^{2}+9 x-5$

$$
a(x) \cdot b(x)=\left(x^{2}+2 x\right)(7 x-5)=7 x^{3}+9 x^{2}-10 x
$$

$$
\begin{aligned}
& a(x)-b(x)=x^{2}+2 x-(7 x-5)=x^{2}-5 x+5 \\
& a(x) \div b(x)=\frac{x^{2}+2 x}{7 x-5}
\end{aligned}
$$

## COMPARING FUNCTIONS

Functions can be compared by their keys features such as minimums, maximums, slopes (rates of change), intercepts, etc. Consider the following three quadratic functions: $p(x)=(x+2)^{2}+4, q(x)=(x-4)^{2}+3, r(x)=x^{2}+1$

COMPARING:
LEAST

GREATEST
$\boldsymbol{r}$-INTERCEPTS
$r: 1$
p: 8
$q: 19$

MINIMUMS
$r: 1$
$q: 3$
p: 4

RATES OF CHANGE ON [0, 1]
$q:-7$
$r: 1$
p: 5

EXAMPLES Use the given functions to answer the following questions.

$$
a(x)=x^{2}+2 x-7 \quad b(x)=-2 x+5 \quad c(x)=x^{2}+4 \quad d(x)=x+3
$$

(1.) Find $a(x)-d(x)$
(2.) Find $b(x) \cdot d(x)$
3. Find $a(x)+c(x)$
(4.) Find $c(x) \div d(x)$
3. List the functions in order from least to greatest by $y$-intercepts.
(4.) List the functions in order from least to greatest by rates of change on $[0,1]$.

Use the given functions to answer questions $1-10$.

$$
\begin{array}{lll}
f(x)=x^{2}+4 x+3 & g(x)=-x^{2}+4 x & h(x)=x^{2}-6 x+2 \\
k(x)=2 x+1 & m(x)=5-x & n(x)=x^{2}-6 x+1
\end{array}
$$

Find the following combined functions. Simplify when possible.

1. $m(x)+k(x)$
2. $m(x) \cdot k(x)$
3. $h(x)-n(x)$
4. $g(x)+n(x)$
5. $f(x) \cdot h(x)$
6. $m(x)+k(x)+n(x)$
7. $m(x) \div k(x)$

Order the specified functions from least to greatest by the given key feature.
8. Functions $f, h$, and $n$ by minimums.
9. Functions $f, g, k$, and $m$ by $y$-intercepts.
10. Functions $g, h, k$, and $m$ by rates of change on $[0,1]$.

A certain company builds computers. The monthly cost for hardware per computer is modeled by the function $H(x)=x^{2}-30 x+300$, and the monthly cost for software per computer is modeled by $S(x)=x^{2}-26 x+200$, where $x$ is the number of computers produced, and $H(x)$ and $S(x)$ are the corresponding costs in dollars.
11. Write a function that models the company's total monthly cost per computer for producing $x$ computers.
12. Use your model to calculate the total cost per computer if 10 computers are produced.
13. How many computers should the company produce in order to minimize the total cost per computer?
14. What is the minimum cost per computer?
15. Using the functions at the top of this page, find $(n(x))^{2}+(m(x))^{2}$

